Problem Set 6

Due **Thursday 5/8/2025**

Note: You are not allowed to use AI assistance for generating solutions or code for these problem sets.

If you are using a python notebook, please submit your python notebook ipynb file with your PSet. Otherwise you must submit your code file in a way that it can be run. Please also submit proof that your code runs properly and yields the desired answers. Finally, please clearly mark what sections of your code are for what problems.

Not following these instructions will result in points deducted for this and all future PSets.

Problem 5-1 [21 points + 2 bonus] Ytterbium nuclear spin qubits
For this problem we will consider long-lived nuclear spin qubits with neutral ytterbium atoms in optical tweezers: Ma et al. (Thompson group), "Universal gate operations on nuclear spin qubits in an optical tweezer array of ^{171}Yb atoms," *Phys. Rev. X* **12**, 021028 (2022).

We will primarily focus on single and two qubit gates (Figures 2 and 3 in the paper). There are also many details that go into trapping individual ytterbium atoms that we will not investigate in depth.

(a) The authors use ^{171}Yb as their atomic species. One motivation is that, in the ground state manifold, the two spin states are nuclear spin states. What is the reason for not having to consider the electronic spin states? Why is this different from alkali atoms such as rubidium and cesium?

(b) Using the nuclear spin of ^{171}Yb as qubit states has both advantages and challenges. Briefly state what, in your opinion, is the biggest advantage and biggest challenge. Explain your reasoning.

(c) What are the wavelengths used for (*i*) trapping, (*ii*) initialization, (*iii*) readout, (*iv*) single qubit manipulations and (*v*) two-qubit manipulations?

(d) Let's now turn to the single-qubit manipulations. Use the numbers presented in the text to simulate Figure 3(a) in QuTiP.

(e) The authors use randomized benchmarking to characterize their gate fidelities. Explain how this technique can separate out errors caused by state preparation and measurement (SPAM errors) from gate errors. Equation 1 from the paper may be helpful in developing this intuition.

(f) What are the main limitations of the coherence time?

(g) The two-qubit gate in the paper is similar to the one you investigated for the last Pset. One difference is that the drives are applied at non-zero detuning. The underlying principle is still the same, in that the states $|01\rangle$ and $|10\rangle$ enclose a different area on the Bloch sphere as compared to $|11\rangle$ (see also Figure 2(b) and (c)). Verify that for $\Delta/\Omega_r \approx 0.377$, $\xi \approx 3.902$ and $\tau\Omega_r \approx 4.293$, this indeed realizes a CZ gate (up to single qubit gates), by simulating in QuTiP.

(h) (Bonus 2 pts) Recreate the trajectories for the different qubit states on the Bloch sphere (Figure 2 (b)), in QuTiP.

Problem 5-2 [15 points] High-fidelity trapped ion gates.
Here we look at the paper by Ballance et al.(Lucas group), "High-fidelity quantum logic gates using trapped-ion hyperfine qubits," *Phys. Rev. Lett.* **117**, 060504 (2016).

The authors demonstrate a high-fidelity implementation of the geometric phase gate, as well as high-fidelity single qubit gates.

(a) The authors use different hyperfine states for the demonstration of high-fidelity single qubit gates and high-fidelity two-qubit gates. Why do they choose these states?

(b) Why do the authors place the ions at a distance of 12.5 wavelengths in the standing wave along the trap axis?

(c) What are the main sources of SPAM errors, and why is it justified to correct for them in Figure 3(a)? In a full quantum algorithm, would these errors matter?

(d) In Figure 3(a), the authors identify two different error regimes for short and long gate times. Please describe these regimes and what limits the gate fidelity in each of them.

(e) If you could change one piece of equipment in this experiment in order to improve the gate fidelity, which one would you choose?