

MENG 31500 - 2026: Practice Questions on 2nd Quantization

March 2, 2026

1. *Basics.* In class, we gave an explicit definition of second-quantized creation operators \hat{a}_j^\dagger for bosonic particles based on how they act on Fock states:

$$\hat{a}_j^\dagger |n_1, n_2, \dots, n_j, \dots\rangle = \sqrt{n_j + 1} |n_1, n_2, \dots, n_j + 1, \dots\rangle \quad (1)$$

Use this definition to confirm that

- (a) $[\hat{a}_i, \hat{a}_j] = 0$
 - (b) $[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij}$
 - (c) $\hat{a}_j^\dagger \hat{a}_j$ is the number operator for orbital j
2. *Bosons in a harmonic oscillator*

Consider a system of bosonic particles in 1D confined in a harmonic oscillator potential. All the particles have the same mass m and feel the same confining potential $V(x) = (1/2)kx^2$. Suppose we prepare the system in the un-normalized state

$$|\psi\rangle = (\hat{a}_1^\dagger)^1 (\hat{a}_2^\dagger)^3 |0\rangle \quad (2)$$

where $|0\rangle$ is the vacuum state with no particles, and \hat{a}_j^\dagger creates a particle in the j th energy eigenstate of a harmonic oscillator.

- (a) Derive the correct normalization factor for this state using second quantization.
 - (b) Write this state out explicitly using first quantization
 - (c) What is the energy uncertainty in the state $|\psi\rangle$?
 - (d) Suppose we focus on just one of the four particles in this state. Is the energy of this particle definite? If not, what is its energy uncertainty?
3. *Two particle stimulated emission*

Imagine we have a set of identical bosons, initially prepared in a Fock state $|N, M_1, M_2\rangle$ having N bosons with wavevector \vec{k} , M_1 bosons with wavevector $(\vec{k} - \vec{q})$ and M_2 bosons with wavevector $(\vec{k} + \vec{q})$. If we now turn on a two-particle potential $V_2(\vec{R}) = 2V_0 \cos(\vec{q} \cdot \vec{R})$ (with \vec{R} being the separation between the two particles), Fermi's golden rule tells us that we will make transitions to the state $|N - 2, M_1 + 1, M_2 + 1\rangle$. This transition rate will involve the physics of stimulated emission for a two particle transition, in that the rate will depend on M_1 and M_2 .

- (a) Calculate the relevant matrix element which goes into Fermi's golden rule for this transition using first quantization (i.e. write out the wavefunctions). You don't have to do any integrals, but explain clearly all the counting arguments involved in getting your answer. Make sure that you properly normalize the initial and final state. (*Hint: just generalize what we did in class for a transition due to a single-particle potential*).
- (b) Calculate the same matrix element using second quantization. Again, be sure to normalize your initial and final states.
- (c) Attempt to provide a heuristic explanation for each factor in your final answer.